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Theory and Applications of Neural Maps

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Abstract. In this tutorial paper about neural maps we review the current state in theoretical aspects like mathematical treatment of convergence, ordering and topography, magnification and others. Therby we concentrate on two well-known examples: Self-Organizing Maps and Neural Gas. Moreover we briefly reflect outstanding applications showing the power of neural maps.

1 Introduction

Fundamentals and basic notations Neural maps occur in real brains in all sensory modalities as well as motor areas being an important step in information processing. From these biological fundamentals models have been derived, which today constitute an important neural network paradigm comprising a broad variety of methods ranging from statistical approaches to strong biologically realistic models [2, 56, 81, 113, 149]. In this paper we restrict ourselves to those models which are designed for data processing. In this technical context, neural maps are utilized in the fashion of topographic vector quantizers. More formally, in neural mapping we consider a set $\mathbf{W} \subseteq \mathbb{R}^{D_V}$ of reference vectors $\mathbf{w_r}$ (codebook vectors) to represent a large data set $V \subseteq \mathbb{R}^{D_V}$. Each vector is uniquely assigned to a certain $\mathbf{r} \in A$ whereby A is an arbitrary index set. Reflecting the biological roots, the elements of A are called neurons. Then, a data vector $\mathbf{v} \in V$ is projected onto that neuron $\mathbf{s} \in A$, of which the reference vector $\mathbf{w_{s(v)}}$ has a minimum distance d to $\mathbf{v}$, compared to all elements of $\mathbf{W}$:

$$\Psi_{V\to A} : \mathbf{v} \mapsto \mathbf{s} = \underset{\mathbf{r}\in A}{\operatorname{argmin}} \left(d\left(\mathbf{v}, \mathbf{w_r}\right)\right). \tag{1}$$

The distance $d(\mathbf{v}, \mathbf{w_r}) = \|\mathbf{v} - \mathbf{w_r}\|$ is based on an appropriate norm, usually the Euclidean. The reverse mapping is $\Psi_{A\to V} : \mathbf{r} \mapsto \mathbf{w_r}$. Both functions together determine the (neural) map $\mathcal{M} = (\Psi_{V\to A}, \Psi_{A\to V})$. All data points $\mathbf{v} \in \mathbf{V}$ that are projected onto the neuron $\mathbf{r}$ make up its (masked) receptive field $\Omega_{\mathbf{r}}$.

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